The resolution of uncertainty in mathematics classrooms

David Clarke and Sue Helme
University of Melbourne
Parkville, Victoria 3052, Australia

This paper addresses negotiation as a social process related to the practices of mathematics (and science) classrooms and associates the need for negotiation with the occurrence of uncertainty. It is our argument that one pathway to knowing is via the resolution of uncertainty, that the process of resolution is fundamentally negotiative, that negotiation is mediated by language, that language presumes intersubjectivity, and that the matter of intersubjectivity is meaning. Data is presented illustrating the role of intersubjectivity as both mediating agent in the resolution of uncertainty, and as product of the negotiative process. Empirical evidence is reported regarding the occurrence of student uncertainty with regard to academic content encountered in classroom settings and the means by which these uncertainties are resolved.

Introduction

The research reported in this paper was undertaken as part of the Classroom Learning Project. This project has as its ultimate goal the modelling of the learning process as it occurs in classroom settings. One immediate task undertaken within the project has been to establish an empirical foundation for the constructs from which a model of learning might be constructed. Several of these constructs are best understood as metaphors: negotiation, abstraction, and mediation, for instance. Each such metaphor carries a burden of associative meaning that may or may not facilitate our understanding of the phenomenon or cognitive object signified by the construct. It is a characteristic of the discourse of the educational researcher that such terms are frequently gerunds (verb/nouns), open to interpretation as both process and product, but having the potential to confuse the distinction between the two. One question that must be asked of any theory employing such terms is whether the construct (abstraction, for instance) is intended to signify an activity (a process) or an outcome (the product of a process).

The phrase "negotiation of meaning" has been used to describe the means by which learning occurs in social settings. Earlier papers (Clarke, 1993, 1994, 1996) have examined the legitimacy with which negotiation is employed to characterise a cognitive process associated with learning. This paper addresses negotiation as a social process related to the practices of mathematics (and science) classrooms and associates the need for negotiation with the occurrence of uncertainty. It must be stressed that negotiation may be triggered by other states besides uncertainty: disagreement, for instance. However, this paper is focussed on documented instances of uncertainty, and the means by which such uncertainties were resolved.

In the analysis reported here, the social goal of resolving uncertainty is taken as the starting point, and the means by which any resolution is achieved is documented. In this account of classroom learning, intersubjectivity enters as a mediating agency, essential to the negotiative process, whereby uncertainty is resolved, and new knowings are constructed. Whereas Voigt's recent analysis starts from the learner's subjectivity and models the establishment of intersubjectivity (Voigt, 1995), this account starts from evidence of students' intersubjectivity and documents alternative forms of uncertainty and the process whereby resolution is achieved; a process in which intersubjectivity has a central role.

Research Methodology

The origins and rationale for the research methods employed in the Classroom Learning Project are outlined elsewhere (Clarke, in press). The current data base for the project consists of videotape records of fifty-five secondary maths and science lessons obtained using two cameras. One camera was directed at the teacher, while the other camera was focussed on a group of about four students. The teacher's utterances were recorded through a radio microphone, and a single microphone was used to record the conversations of all four students. The two video images were mixed on site to produce a composite picture in which the students occupy most of the screen with the teacher image superimposed in a corner of the screen. This

combined image was recorded onto video-8 tape using a compact videorecorder attached to a laptop computer. The researcher, seated at the rear of the classrom, was able to listen simultaneously to both student conversations and teacher utterances and to record field notes onto a word processing document on the computer. The field notes were "time-tagged" to corresponding events in the video record using *CVideo* software (Roschelle, 1992). The field notes enabled the researcher to document impressions of significant classroom episodes and learning events as well as to provide reference markers for the subsequent interviewing of student subjects. The research procedure was developed in an attempt to study learning in legitimate classroom settings, while minimizing the need for researcher inference regarding student thought processes. To infer student thought processes and the significance of classroom events on the basis of videotape data only seems an unjustified extrapolation. The video-stimulated recall interview has become an accepted research technique (see, for example, Anthony, 1994), and technological advances such as *CVideo* further enhance its effectiveness. On-site production of the linked field notes and split-screen video record allow the researcher to conduct such student interviews immediately after the lesson being studied.

The videotape record was used in the interview to stimulate student recall of classroom events. The use of the *CVideo* software enabled the researcher to locate within the field notes reference to actions of the student which seemed to be of significance either to the researcher or to the student. Having found a particular item in the word document, the software could be used to find very quickly the corresponding moment on the video record. This was then played back and discussed. In such interviews, students reconstruct their motivations, thoughts and actions, prompted by the video record of the classroom events, in full knowledge of the ultimate outcomes of those actions. In this situation, the reconstructed account provides both an explanatory interpretation of the video record and a reflective commentary on the relationship between action and outcome. The audio record of the interview provided a third source of data.

For the purposes of the analyses reported in this paper, six "integrated data sets" were generated from three mathematics and three science lessons. Each integrated data set is a single text document incorporating transcriptions of videotape and interview dialogue together with the researcher's field notes, time-tagged to the video record, and supplemented by copies of student written material. Instances in which there was evidence of student uncertainty were identified and these episodes were then categorised according to the nature of the uncertainty and the means (if any) of resolution.

Uncertainty and Resolution

It is our argument that one pathway to knowing is via the resolution of uncertainty, that the process of resolution is fundamentally negotiative, that negotiation is mediated by language, that language presumes intersubjectivity, and that the matter of intersubjectivity is meaning. Before presenting our empirical data, it is essential that we establish a theoretical framework with regard to each of the constituent terms in the preceding sentence.

Negotiation
Negotiation has been characterised in some detail elsewhere as a cyclic process of refraction (construal), reflection, and representation, the goal of which is consensus (Clarke, 1996) Cobb and Bauersfeld define the negotiation of meaning succinctly as "the interactive accomplishment of intersubjectivity" (Cobb & Bauersfeld, 1995, p. 295). Lave and Wenger, in associating learning with participation in practice, assert that "Participation is always based on situated negotiation and renegotiation of meaning in the world" (Lave & Wenger, 1991, p. 52) Since classrooms represent legitimate sites of situated mathematical practice, this perspective supports the need for the empirical documentation of negotiative processes in the classroom.

Meaning
The presumptions of meaning are community, purpose and situation. It is futile to discuss the meaning of a word or term in isolation from the discourse community of which the speaker claims membership, from the purpose of the speaker, or from the specific situation in which the word was spoken. Indeed, it is not the word that has meaning, but the utterance. Our understanding of meaning has been usefully informed by consideration of the writings of Bakhtin and his colleagues and interpreters.

In all these cases, we are dealing not with the isolated words as a unit in language, nor with the *signification* of this word, but with the completed utterance and its concrete *meaning*, the content of this utterance (Bakhtin, 1979, quoted in Todarov, 1984, p. 53). Since the data with which we work consist of utterances of one form or another, and since the focus of this paper is the resolution of uncertainty, Baktin's statement that "Meaning always answers some questions" (Bakhtin, 1979, quoted in Todarov, 1984, p. 54) is particularly useful. It identifies meaning with the need to know and, by implication, with the resolution of uncertainty.

The Role of Intersubjectivity

Language is constitutively intersubjective (Todarov, 1984, p. 30). As noted in the introduction to this paper, a level of student-student and student-teacher intersubjectivity is prerequisite to the negotiative processes by which the resolution of uncertainty is attempted. The role of intersubjectivity in negotiation can be argued successfully from the dependency of negotiation on language (or at least on some form of communicative process), but this does not excuse us from the obligation to present empirical evidence of intersubjectivity in action. An understanding of the means by which the resolution of uncertainty might be achieved requires an understanding of intersubjectivity as a phenomenon of social interaction. To establish this point, consider the following transcript (All utterances are by students. K and L were subsequent interviewees, S19 and S20 were not):

Episode 1

Line	Transcript
1	S19: It says how many sheets of graph paper would you need to show one million one millimetre squares.
2	L: To show one million, you know you don't divide it by one hundred, because there's more than a hundred one millimetre squares. I mean you're going to find the area of this.
3	K: What?
4	L: You've got to find the area of this, there's more than one hundred one millimetres.
5	K: That's right. I was doing length by—oh screw that.
6	L: One hundred one millimetre squares.Take length—
7	K: Um, there's how many down here?
8	L: And along that side there is—
9	K: How many are there down there?
10	L: There's a hundred one millimetres there.
11	L: No, there wouldn't be.
12	K: There wouldn't be, that's not right.
13	L: It'd be 250.
14	K: Yeah.
15	L: Yeah, there'd be 250, length of graph.
16	K: OK, so it would be length times width [inaudible]
17	L: And uh, 250 millimetres. Width—
18	K: What's width?
19	L: That's—
20	K: That's 10, 20, 30, 40, 50—
21	L: 18, 180.
22	K: Times 180. OK here we go. 250 [inaudible] 45 thousand.
23	K: Equals 45 thousand.OK, that's 45 thousand millimetres. What's a million divided by 45 thousand and times it by that?
24	L: OK. Therefore there are 45 thousand mm squares.
25	S20: 45 thousand mm?
26	L: Yeah.
27	S20: 45 thousand.
28	K: 22.2.
29	L: Sheets. Of graph paper.

Evidence for the enactment of intersubjectivity should assist us in locating intersubjectivity within the learning process as either agency or as outcome or as both. In the above transcript, there are several indicators of intersubjectivity. First, much of the actual text is incoherent as written text: sentences are ungrammatical or incomplete; pronouns are used without textual clues as to their referrents; single word utterances are frequent. Communication in this form can only be sustained if the participants share understandings of the referrents of the pronouns or key words, and of the processes, actions, or relationships suggested (but not stated) by the sentence fragments and the participants' gestures. Further evidence of intersubjectivity can be seen in the manner in which one speaker will complete the sentence of the previous speaker, as occurs in lines 28 and 29 (we are using "line" as a shorthand for "utterance"). In this sense, it is useful to distinguish the overt text in the form of the literal transcription from the implicit text being co-constructed by the participants, the existence of which can only be inferred on the basis that the interaction appears to have been both purposeful and successful.

Identifying Uncertainty

We are employing "Uncertainty" to refer to a behavioural state. Examples are provided below regarding several forms of student uncertainty with regard to academic content encountered in classroom settings for which we have empirical evidence.

- Uncertainty regarding a *fact* or correct term;
  ["Why is it V, what does V have to do with speed?"]
- Uncertainty regarding the correct form of an action or *procedure*;
  [D: Can you tell me how to work that one out?
  M: You get eight four oh which is the number of students, divide five, times two.]
- Uncertainty as to the *meaning* of a term or an activity;
  ["How can the graph go below zero? You can't have a minus ten *speed*"]
- Uncertainty regarding a form of *representation*;
  ["It's the velocity, not the way it bounces"]
- Uncertainty as to the *correctness* of the student's opinion, understanding, method, or outcome.
  ["Tell me if you think this looks sort of logical"]

This categorization of Uncertainty is unlikely to be exhaustive with regard to all possible classsroom settings or situations. Rather, the categories provide an adequate classification of our existing data set.

Resolution

Resolution is yet another term open to interpretation as either process or product. The categorization system, that we have employed in this paper, is based on the dominant authority appealed to in the *process* of Resolution. For the purposes of this discussion, Resolution is taken to involve one of four forms, but these are not mutually exclusive, and a particular episode may involve student appeal to several authorities in seeking resolution of the same uncertainty.

Resolution via an appeal to *empirical evidence*;
Resolution via an appeal to a *person* (a knowledgable other, such as the teacher or another student);
Resolution via an appeal to a *text*;
Resolution via an appeal to *prior experience*.

An adequate characterisation of Resolution as outcome would require a more extensive data base and analysis than has been possible to date, and as such is beyond the scope of this paper. It is possible, however, within the scope of this paper, to anticipate and even to document some likely associations between Uncertainty and Resolution. Since, theoretically, each of the categories of Uncertainty may be resolved by any of the above means of Resolution, it is possible to consider the interaction of Uncertainty and Resolution. That is, does the knowing which is prompted by student uncertainty as to a procedure and which is resolved by appeal to a peer differ in a significant manner from the knowing which is prompted by student uncertainty regarding a representation and resolved by reference to a text (for example)? We conjecture that these differences might well be profound.

Results And Discussion

Examination of the six integrated documents revealed twenty-eight classroom "episodes" in which one or more students expressed uncertainty. The episodes comprised the dialogue and activities that students engaged in as they approached, worked on and reached completion of a particular classroom activity, such as a problem solving task. Thus, each episode was a coherent unit of activity unified by a single purpose. Within each episode there were identifiable "negotiative events" consisting of a number of utterances—questions, statements, answers—where a particular form of uncertainty was expressed and sometimes resolved. For example, a string of utterances relating to uncertainty about a mathematical procedure may be resolved by interaction with peers and teacher as a student participates in classroom activity. Forms of associated uncertainty and resolution are illustrated in more detail in the example below.

Uncertainty as to procedures and outcomes resolved by appeal to peers.
In this episode, year 8 students had been asked to find the height of a stack of one million sheets of paper. Uncertainty about procedures and outcomes was resolved through peer interaction. Appeal to peers was in frequent evidence in our data and the following episode is a particularly rich example of the features of this form of classroom negotiation (K and L are student interviewees; S20 and S22 student non-interviewees; and, T is the teacher).

Episode 2

	Line	Transcript
E V E N T 1	1	L: [writing] 500 sheets. Height equals.
	2	K: OK, question 2 [Find the height of a stack of one million sheets of paper].
	3	L: Does everyone understand what we did with number one?
	4	K: No, but. Anyway. 500 sheets.
	5	L: And how many sheets do we need?
	6	K: 500 sheets of what? 500 sheets.
	7	L: Their height equals 5.8.
	8	K: I know. But we've got to do it all together so.
	9	L: One point oh times ten to the power of 6 divided by 500.
	10	K: Oh yeah sure what are you talking about? What are you talking about?
	11	L: Finding out how many five hundreds there are in a million.
	12	K: How many five hundreds there are in a million. That would make one thousand. How many thousands are there in a million? That would make a thousand, two thousand. What? [to another S]. I have a lot to say. This is called skill. This is what you do. You divide 500 into a 100 which is 2. Then you do a hundred [correcting herself] A thousand.
	13	L: Times five point eight. Shush.
	14	K: Which is 2. Then you do a thousand into a million which is 1000, so 1000 times 2 is 2000.
	15	L: Eleven thousand six hundred.
E V E N T 2	16	T: With your working out folks I want you to tell me what you are mult— Matthew— what you are multiplying by, and you simply put a little arrow telling me what and why.
	17	K: What are we doing? Is it a million sheets of paper though?
	18	L: Yeah. One point—
	19	K: We're doing a million sheets of paper.
	20	L: Yeah, you need—
	21	K: Yes we do. We do, shut up.
	22	L: Therefore, I did that wrong.
	23	K: 2000 times 5.8 is eleven, six, zero, zero.[ie 11,600]
	24	L: And then?
	25	K: Which means—which could make it 11 point 6 metres, right?

Episode 2 (continued)

	Line	Transcript
E V E N T 3	26	L: Or on the - yeah. 'Cause you take off one to get the centimetres, and another one, yeah.[pause] All right. And you've got to point out what the units are, right?
	27	K: You've got to point out the what is?
	28	L: You have to show what to multiply too.
	29	[S22 says something to K, K laughs]
	30	S20: That's not how you know, you look like you know what you're doing and you just do it.
	31	K: Exactly, you go into a state of total concentration, it lasts about 2 seconds, that's when you get the answer, and then you don't know what you're doing, so it doesn't matter. 500 sheets equals point, 5 point 8 centimetres. I don't even understand what I wrote.[pause]
E V E N T 4	32	K: Why do you divide a million by 500 to get that answer?
	33	L: Because you know, you know what the height is.
	34	K: Tell me what I'm doing here, tell me what I've done.
	35	L: All right. You know that 500 sheets equals 5.8 centimetres. Oh, I can't see it.
	36	K: Get some other glasses.
	37	L: Now, we need to know - we need to know the height of a million sheets of paper. Therefore you must divide a million by 500 and times that number by 5.8.
	38	K: [writing] Equals 2 thousand. Sheets of paper. OK, 2000 times 5.8 centimetres equals 11,600 centimetres, equals 11.6 metres of paper.[bell rings]

The above interaction exemplifies several features of negotiative exchanges between peers:

- The occurrence of "negotiative events" within an "episode"
  Event 1 combines the refinement of intersubjectivity within the group with L's first solution attempt (1 to 15).
  Event 2 revisits the procedure employed in Event 1 (16 to 25).
  Event 3 involves the negotiation of appropriate units of measurement (26 to 31)
  Event 4 reviews the procedure again and links it to the task (32 to 38)
- Student construal of the requirements of the didactic contract
  (Brousseau, 1986) (Line 8)
- Self-correction (a form of internalised negotiation) (Line 12)
- Completion by one student of a statement commenced by another student (12 and 13)
- Review of purpose and restatement of task (17 and 19)
- Regulatory or organizational interaction (Lines 3, 13 and 21)
- Evidence of intersubjectivity as mediating agency (Line 26)
- Student self-evaluation (metacognition) (Lines 12, 17, 22, 31, 34)

The transcript above may be considered as one episode in a lesson consisting of many episodes. An episode is defined by a consistent purpose such as the solving of a particular problem. Each such episode may involve several negotiative events. A negotiative event is defined by an identifiable intermediate purpose; a purpose whose realization is an intermediate goal within the encompassing episode. Each negotiative event may be composed of several utterances, each with its own immediate purpose. An understanding of a given episode requires an understanding of each constituent level of negotiation: the negotiative event and the utterance. Several other examples could be provided, but it is hoped that Episode 2 serves as an appropriate illustration both of the uncertainty-resolution relationship as it is enacted in classroom settings, and of the analytic procedure being employed.

Resolution not achieved

There were a number of instances where students were not able to resolve their uncertainty within the time allowed by the lesson or in the interview following that lesson. Not surprisingly, more superficial forms of uncertainty (such as procedural uncertainty) were usually cleared up within the lesson. Uncertainty about more complex and difficult material was less readily resolved.

Conclusions

It is a preliminary goal of the Classroom Learning Project to provide an empirical base for the constructs in current use in our theorizing regarding classroom learning. As the body of empirical data grows, we begin to see the nature of the constructs which our theories of learning must employ, and we begin to see the relationships between these constructs. The data reported in this paper are intended to contribute to our understanding of intersubjectivity as both an outcome of classroom negotiation and as a mediating agency in the resolution of student uncertainty in mathematics classrooms. This paper reports evidence for the relationship between negotiation and intersubjectivity as associative, with the role of intersubjectivity identified as both a prerequisite for the classroom negotiation of meaning and the means by which the negotiative process is sustained and social closure of classroom activity signalled through group acknowledgement of the resolution of uncertainty.

It has been demonstrated that the resolution of uncertainty in mathematics classrooms can have a negotiative character. Student classroom activity within a single lesson can be interpreted as occurring in a hierarchy of levels of purpose: the lesson, the episode, the event, and the utterance, where each level can be associated with a specific purpose. Other data, not reported here, documents the purposeful resolution of student uncertainty over several lessons, and even over several weeks. Resolution over such periods of time goes beyond the confines of classroom activity and is not considered in this paper. The resolution of student uncertainty within the classroom can provide the pretext for purposeful activity for at least two of the proposed levels: the episode, and the event, and this has been demonstrated empirically in this report.

It is our suggestion that that the resolution of uncertainty in relation to academic content represents one form of learning occurring in classroom settings. While the resolution of uncertainty is only one of many perspectives that may inform our analyses of classroom learning, it not only provides insight into the negotiative process in operation in the classroom, but suggests at least two areas for further investigation. First, the nature of the cognitive objects which are the focus of student uncertainty, and, second, the forms of knowing that can be associated with particular uncertainty-resolution combinations. A further area for investigation is the tracking of the resolution of uncertainty concerning more complex and difficult academic content over longer periods than those addressed in this paper. It is intended that all of these matters will be addressed in subsequent papers.

Acknowledgements

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